African Journal Of Business And Management
Special Issue: Volume 8, Issue 1, February 2023 http://aibumaorg.uonbi.ac.ke/content/journal
Pgs 347-358

University of Nairobi
School of Business

AFRICAN JOURNAL OF BUSINESS AND MANAGEMENT

(AJBUMA)

ISSN 2079-410X

SIZE AS A FUNCTION OF PORTFOLIO OPTIMIZATION

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Date Received 21/09/2022 | Date Accepted 13/02/2023

Abstract

The issue of the optimal number of securities an investor can hold to achieve optimal diversification has remained controversial in corporate and academic cycles. Harry Markowitz model initiated a novel explanation on the effect of number of securities on portfolio diversification. Modern portfolio theory postulated that increasing the size of a portfolio reduces idiosyncratic risk and the investor can achieve optimal risk-adjusted performance. Markowitz (1952) formulated the portfolio selection problem as a comparison of mean and variance of a portfolio of assets. The number of securities to invest, the combination of securities in a portfolio and the risk involved are vital considerations for investors. Holding too few stocks exposes the investor to idiosyncratic risk while holding too many stocks becomes too costly in terms of numerous transactions required to build initial portfolio and the opportunity cost to monitoring a large diversified portfolio. It is not clear how many stocks are sufficient to achieve full diversification.

***Key words**: Portfolio, Optimization, Size, Risk, Securities.*

Introduction

The discourse on the number of financial assets one should hold in a portfolio in order to maximize the returns at manageable risk levels is attracting attention in both the industry and the academia environments. Scholars have made deliberate efforts to guide the discussion, however the opinion are quite contradicting on what should constitute an optimal portfolio. Therefore the debate is far from over but the researcher in this study has made yet another deliberate attempt to address this issue.

Some scholars have argued that the asset combination depends on the nature and extent of inter correlation among the individual assets involved. It's portended that negatively correlated assets have minimal risks as opposed to positively correlated ones. Modern Portfolio Theory is an investment decision approach which helps investors to classify, estimate and control the kind and mount of expected return and risk. It quantifies the relationship between risk and return and the fact that investors must be compensated for undertaking the risk. It emphasizes on the statistical relationships among different individual securities that comprise the overall portfolio (Edwin & Martins 1997). The MPT formulates mathematically the concept of diversification in investing, with the endeavor of choosing a collection of investment assets that collectively have a lower risk than any individual asset.

Zhang, Peng and Li (2015) discussed the selection problem in uncertain environment where security returns cannot be clearly reflected by historical data but by evaluation of experts. Kara, Ozman and Weber (2019) attempted to examine the portfolio optimization problem in situations of uncertainty while Solares et al (2019) delved into the portfolio optimization challenge in scenarios of uncertainty proposing an approach of confidence intervals. All these scholars assumed that investors are rational and the market is efficient with a total disregard of individual investor differences. They belief that investors have homogeneous expectations and consistent attitudes towards risk. However in reality there are many anomalies in these assumptions based on empirical studies on financial markets for example stock premium, effect of small firms, investors going for insurance among others.

Further studies on investor behavior reveals that investors are not completely rational but exhibit irrational behavior such as cognitive bias, over confidence and selection preference. It has been found that investors set a reference point in advance whenever making investment decisions and the appetite for risk revolves around the reference point (Kahneman and Tversky 1979) hence cognitive psychologists have suggested prospective theory.

Literature Review

Modern Portfolio Theory

Harry Markowitz (1952) propounded the Modern Portfolio Theory by initiating a novel description of the effect of diversification on portfolio risk. In his dissertation he argued that portfolio risk can be reduced through diversification. The theory made an attempt to maximize portfolio expected return by cautiously selecting the proportions of various financial assets to include in the portfolio (Rani 2012). Mangram (2013) portend that MPT is a combination of Markowitz portfolio selection theory and Capital Asset Pricing Model which was contributed by William Sharpe in 1964.

Modern Portfolio Theory is an investment framework for selecting and constructing of investment portfolios through maximization of expected returns and minimization of risk of a portfolio (Fabozzi, 2002). Markowitz theory of portfolio selection is a normative theory that defines a standard behavior that investors should purse in constructing a portfolio. On the other hand Sharpe asset pricing theory hypothesizes how investors actually behave in contrast to the norms.

MPT assumes that investors are rational and the markets are efficient. For a given amount of risk, the MPT shows how an investor can select a portfolio with the highest possible expected return. Equally for a specific expected return, the MPT explains how to select a portfolio with the lowest possible risk (Gruber, 2011). MPT models an asset return as a normally distributed function, defines risk as the standard deviation of the return and portfolio.

The Markowitz model of mean-variance requires determination of composition of a portfolio of assets that seeks to minimize risk at a predetermined level of expected return (mean). The theory of mean variance is based on the presumption that rational investors select from a list of risky assets purely on the basis of expected return and risk.

The current pitfall of portfolio optimization is due to error maximization (Michand 1989, Nawarocki 1996). Equally variables informing mean-variance optimization suffer from uncertainty challenge and the procedure chooses assets with most appealing characteristics. This makes the mean-variance model subject to estimation error which is the highest likely error. Further the impact of the estimation error on portfolio weights magnifies the estimation error problem. In addition the market evolution maximizes the estimation error issue since the past is not representative of the future market behavior.

It has been proposed by some scholars (Michand and Michand ,1998) that Monte Carlo based procedure for dealing with sampling error or sampling estimation error can be adopted to solve the estimation error issue. Black and Litterman 1992 argued that Bayesian approach is suitable in solving non-stationary estimation error.

A part from Markowitz (1952), other subsequent scholars have proposed various models such as Mean- Value at Risk and Mean Conditional value at Risk. Markowitz used probability theory to deal with uncertainty of the investment. In reality though, there are many uncertainties in the financial markets for instance, change of government policies, internal macro economic factors among others. Therefore there is no reference data especially for the emerging securities. The uncertainty theory proposed by Liu BD (2007) and Liu JJ(2004) has attempted to address a new hybrid intelligent algorithm to address this uncertainty of returns optimization problem. Wang and Chen (2019) also weighed in on determination of mean-conditional value at risk as an alternative model to MPT.

The Mean Variance model seeks to achieve the minimum risk with certain expected rate of return constraint and to maximize the rate of return under specified maximum risk which can be torelated. Bander, Bodnar, Parolya and Schmid (2020) in their article Bayesian mean-variance analysis; optimal portfolio selection under parameter of uncertainty solved the portfolio selection problem when parameters of mean vector and covariance matrix are not known and historical data is being used for estimation. The Bayesian posterior distribution was used as an observable sample that is using future realization distribution of asset returns as an observable sample.

Capital Asset Pricing Model (CAPM)

CAPM states that the aggregate market portfolio happens to be efficient. The theory further affirms that since investors can eliminate firm specific risk by diversifying their portfolios no investor is allowed to attach a price to it. Therefore CAPM asserts that all risks originates from a single factor, the market. These systematic risks cannot be diversified to eliminate them. Therefore the expected return of any asset is determined by its co variation to the market. Fama and French (2004) portend that suitability of CAPM as a single factor model is on its simplicity since it offers a quantitative view into risk-return interplay and a valuable tool for predicting expected returns.

In the converse the empirical validity has been widely criticized especially in the academic cycles due to its idealized assumptions. Levy and Roll (2010) however, argue that slight adjustments with estimation error in relation to the sample parameters used in evaluation of market portfolio suffice in making the market proxy efficient. This acts to support the CAPM concept. They further add that validity of the global CAPM is not empirically testable or verifiable as the true market portfolio is in essence not observable thus occasioning the use of proxies.

Many scholars object the view that market alone is not sufficient in explaining assets returns (Banz 1981; Basu 1983). Rose 1976 derived an asset pricing model postulating that expected returns of an asset is influenced by several risk factors. This argument is more universally acceptable. The Arbitrage Pricing Theory is less restrictive in its assumptions especially the assumption that investors are mean-variance optimizers. However its main weakness is that it does not specify the factors to include in the model. Therefore in as much as it is a strength to allow multiple factors, it at the same time a weakness since its not specific as to the factors to include in the model.

Fama and French three factor model is an extension of CAPM (Fama and French 1995). The scholars proposed inclusion of covariance of returns; co variance of book-to market value of stocks in addition to the market factor proposed by CAPM. They argued that there are unidentified variables which produce systematic risks in returns.

The Black –Litterman Model

In the realm of asset allocation this model is an extension of MVO. Black and Litterman (1992) originates from the equilibrium assumption that global portfolio is well diversified and efficient (CAPM). Therefore using reverse optimization process the model derives returns of assets which are implied by the market equilibrium. The model combines market equilibrium and extra market views of the investor. This model ensures that estimation error is reduced in the final portfolio and portfolio weights are more intuitive in respect to the investor views and optimized portfolio is more efficient and less concentrated towards single assets. The biggest limitation of this model is the inclusion of investors' views in the model. These views should be included with extra caution else the model collapses. The levels of confidence in the views expressed is not discussed by Black and Litterman (1992)

Michand et al (2012) argue that errors in the covariance matrix are likely to overweigh the optization process especially when the number of assets increases. Co variance in the context of monetary value optimization is estimated through employing sample covariance matrix. This is likely to through the results to a sampling error because it is based on the data as contained in the sample. The question as to which data remains sufficiently representative to accurately reflect the true state of the covariance returns remains begging.

Research Methodology

This study employed positivism research philosophy because it permits the use of probability and deductive logic in deriving meanings of situations and allows a scientific analysis of the data.

Kothari(2009) argued that research design is an arrangement of conditions for collection and analysis of data in a way to combine relevance of research purpose with procedure.. Quantitative research design was used to derive the requisite output model from the relationships between the variables used in the study (Saunders & Thornhil, 2007). Target population comprised all the 66 firms listed in Nairobi Securities Exchange from all sectors of the economy. The study covered a 12 month period of September 2021 to August 2022. The stock prices were obtained from the data bases on stock prices as summarised by NSE reports.

Volatility of the data was covered by utilizing the highest and lowest price limits for the various securities traded within the study period.

Data was analyzed using descriptive statistical methods of mean, percentages, and standard deviation, variance and co-variance matrices. Results were presesnted in form of tables.

Ideas and concepts borrowed from various scholars were adequately acknowledged both in the text and in the reference list. Equally, the research findings were published in internationally peer reviewed journals for accessibility to interested parties.

The Model

The rate of return was calculated as a percentage of the increase or decrease in the investor's wealth associated with holding the stock for the period. In this case it was assumed that any dividends were re-invested in the stock. Therefore the annual returns were calculated as

$$R_{it} = \frac{P_{it+1} - P_{it},}{P_{it}}$$

Where;

P_{it} is the stock price at time t; P_{it+1} is the stock price at end of time t; R_{it} is the rate of return on stock i at time t

Stock variance was derived as

$$\sigma^2 R_{it} = \frac{\Sigma (R_{it} - ER_{it})^2}{n-1}$$

Where;

ER_{it} if the average rate of return for stock i in time t ; $\sigma^2 R_{it}$ is the variance of the stock i in time t

In this case it was take as the average of the high and low prices of the respective stocks

NSE Portfolio Return

In this paper the NSE portfolio comprise 57 Shares of all the NSE listed companies which showed a deviation in the prices during the 12 month study period. It was assumed that an investor is interested in all the 57 shares and the researcher measured the effect of reducing the number of assets in the portfolio and how that reduction impacts on optimization.

NSE Portfolio Variance

Assume an equal distribution of the investor's wealth on the 57 shares. There each share takes 1/57 of the investor's wealth. The variations are considered to be the standard deviations of the returns

Sharpe Ratio was also used to measure optimality of the portfolios

$$\text{Sharpe Ratio} = \frac{Rp-Rf}{\sigma p}$$

Results and Discussion

The researcher sought to determine the optimal size of assets in a portfolio. The approach included shares for all firms listed in the NSE and delineated them based on mean –variance criterion by Markowitcs portfolio theory.

Determination of Stock variances and Return Rates

Table 1: Stock Variances and Return Rates

FIRM	Low-StockPrice	High-StockPrice	Stock Variance	Return-Rate
EGAD	11.40	11.50	0.10	0.877192982
KUKZ	365.00	450.00	85.00	23.28767123
KAPC	80.00	110.00	30.00	37.5
LIMT	285.00	507.00	222.00	77.89473684
SASN	17.90	22.60	4.70	26.25698324
WTK	119.00	157.00	38.00	31.93277311
CGEN	15.30	70.00	54.70	357.5163399
ABSA	9.02	12.95	3.93	43.56984479
BKG	24.30	40.00	15.70	64.6090535
COOP	10.30	14.00	3.70	35.9223301
DTK	48.00	67.50	19.50	40.625
EQUITY	38.75	55.00	16.25	41.93548387
HFCK	2.80	5.20	2.40	85.71428571
IMH	16.50	23.40	6.90	41.81818182
KCB	34.00	50.25	16.25	47.79411765
NCBA	23.00	29.00	6.00	26.08695652
SBIC	82.00	108.00	26.00	31.70731707
SCBK	121.00	148.75	27.75	22.9338843
EVRD	0.67	1.27	0.60	89.55223881
XPRS	2.70	4.75	2.05	75.92592593
LKL	2.71	4.40	1.69	62.36162362
NBV	2.68	8.30	5.62	209.7014925
NMG	15.40	26.50	11.10	72.07792208
SMER	2.18	4.38	2.20	100.9174312
SGL	11.50	19.70	8.20	71.30434783
TPSE	10.50	18.00	7.50	71.42857143
UCHM	0.16	0.29	0.13	81.25
SCAN	2.82	6.10	3.28	116.3120567
BAMB	32.00	40.00	8.00	25
CRWN	25.35	44.15	18.80	74.1617357
CABL	0.80	1.62	0.82	102.5
PORT	6.00	9.40	3.40	56.66666667
KENGN	3.35	5.10	1.75	52.23880597

KPLC-P4	4.10	4.53	0.43	10.48780488
KPLC	1.30	2.45	1.15	88.46153846
TOTL	20.65	27.50	6.85	33.17191283
UMEME	5.50	8.48	2.98	54.18181818
BRIT	5.14	8.78	3.64	70.81712062
CIC	1.80	3.15	1.35	75
JUB	240.25	375.00	134.75	56.08740895
KNRE	1.92	2.65	0.73	38.02083333
LBTY	4.91	9.24	4.33	88.18737271
SLAM	9.64	15.00	5.36	55.60165975
CTUM	8.00	18.00	10.00	125
HAFR	0.30	0.42	0.12	40
OCH	1.76	2.70	0.94	53.40909091
TCL	0.90	1.49	0.59	65.55555556
NSE	6.50	10.90	4.40	67.69230769
BOC	62.00	98.00	36.00	58.06451613
BAT	400.00	512.00	112.00	28
CARB	10.10	13.50	3.40	33.66336634
EABL	110.00	180.75	70.75	64.31818182
FTGH	1.08	1.40	0.32	29.62962963
UNGA	27.00	36.40	9.40	34.81481481
SCOM	23.00	45.25	22.25	96.73913043
FAHR	5.00	7.48	2.48	49.6
GLD	1780.00	2135.00	355.00	19.94382022

Market Standard Deviation	51.8698724
Market Portfolio Mean	63.78647113
Sharpe Ratio	1.229740275

The findings indicate that the market portfolio had an expected return of 63.7865% and a deviation of 51.5698 with a Sharpe ratio of 1.2507. The market is thus considered a poor portfolio because the risk is high and the return is low contravening the concept in portfolio theory. A risk taking investor will have motivation for such investment however a risk averse investor would not undertake such an investment.

Delineation of Assets in the Portfolio

Table 2 Delineating Maximum Return and Minimum Variance Portfolios

No.	FIRM	Low-Stock Price	High-Stock Price	Max Return Standard Deviation<=51.5698	Max Return-Rate>=63.7865%
1	BKG	24.30	40.00	15.70	64.6090535
2	HFCK	2.80	5.20	2.40	85.71428571
3	EVRD	0.67	1.27	0.60	89.55223881
4	XPRS	2.70	4.75	2.05	75.92592593
5	NBV	2.68	8.30	5.62	209.7014925
6	NMG	15.40	26.50	11.10	72.07792208
7	SMER	2.18	4.38	2.20	100.9174312
8	SGL	11.50	19.70	8.20	71.30434783
9	TPSE	10.50	18.00	7.50	71.42857143
10	UCHM	0.16	0.29	0.13	81.25
11	SCAN	2.82	6.10	3.28	116.3120567
12	CRWN	25.35	44.15	18.80	74.1617357
13	CABL	0.80	1.62	0.82	102.5
14	KPLC	1.30	2.45	1.15	88.46153846
15	BRIT	5.14	8.78	3.64	70.81712062
16	CIC	1.80	3.15	1.35	75
17	LBTY	4.91	9.24	4.33	88.18737271
18	CTUM	8.00	18.00	10.00	125
19	TCL	0.90	1.49	0.59	65.55555556
20	NSE	6.50	10.90	4.40	67.69230769
21	SCOM	23.00	45.25	22.25	96.73913043

Portfolio Mean or Expected Return	90.13848033
Portfolio Standard Deviation	31.98781907
Portfolio Sharpe Ratio	2.817900156

From the delineated table 2 the portfolio reduced from 57 companies to 21 firms using the Markowitcs criteria of mean-variance based on the market portfolio.

The reduced Portfolio of 21 assets returned a mean of 90.1384 with a standard deviation of 31.9878 and an improved Sharpe ratio of 2.8179. This results posted a better performance than the market portfolio and fitted the argument in the portfolio optimization theory to the effect that portfolio on an efficient frontier will always outperform all other portfolios including the market portfolio. The optimal Markowicks portfolio is moves closer to the minimum variance portfolio from the maximum return portfolio.

Table 3: Delineating for max return at SD<=31.9878

No.	FIRM	Low-StockPrice	High-StockPrice	Max Return Standard Deviation<=31.9878	Max Return-Rate>=63.7865%
1	BKG	24.30	40.00	15.70	64.6090535
2	HFCK	2.80	5.20	2.40	85.71428571
3	EVRD	0.67	1.27	0.60	89.55223881
4	XPRS	2.70	4.75	2.05	75.92592593
5	NBV	2.68	8.30	5.62	209.7014925
6	NMG	15.40	26.50	11.10	72.07792208
7	SMER	2.18	4.38	2.20	100.9174312
8	SGL	11.50	19.70	8.20	71.30434783
9	TPSE	10.50	18.00	7.50	71.42857143
10	UCHM	0.16	0.29	0.13	81.25
11	SCAN	2.82	6.10	3.28	116.3120567
12	CRWN	25.35	44.15	18.80	74.1617357
13	CABL	0.80	1.62	0.82	102.5
14	KPLC	1.30	2.45	1.15	88.46153846
15	BRIT	5.14	8.78	3.64	70.81712062
16	CIC	1.80	3.15	1.35	75
17	LBTY	4.91	9.24	4.33	88.18737271
18	CTUM	8.00	18.00	10.00	125
19	TCL	0.90	1.49	0.59	65.55555556
20	NSE	6.50	10.90	4.40	67.69230769
21	SCOM	23.00	45.25	22.25	96.73913043

Shares for all the 21 firms were retained in the portfolio since their variance were all less that the delineating threshold.

Table 4: Delineating for max return of =>90.1384 with Min SD

No.	FIRM	Low-StockPrice	High-StockPrice	Max Return Standard Deviation<=31.9878	Max Return-Rate>=90.134%
1	NBV	2.68	8.30	5.62	209.7014925
2	SMER	2.18	4.38	2.20	100.9174312
3	SCAN	2.82	6.10	3.28	116.3120567
4	CABL	0.80	1.62	0.82	102.5
5	CTUM	8.00	18.00	10.00	125
6	SCOM	23.00	45.25	22.25	96.73913043

Portfolio Mean	125.195
Portfolio SD	42.74223
Sharpe Ratio	2.929071

Delineating the Sharpe Ratio Portfolio

The market Sharpe Ratio was 1.22974. Therefore all assets whose Sharpe Ratio is less than 1.2297 should be removed from the list. This ratio represents the expected return per unit risk and therefore the portfolio with the maximum Sharpe ratio gives the highest expected return per unit risk and this is the most risk efficient portfolio

The foregoing analysis shows that the market Sharpe was the lowest at 1.2294 with a portfolio of 57 assets; upon reducing the portfolio to 21 assets the Sharpe ratio improved to 2.8179 and further reduction of the portfolio to 6 most efficient assets by mean and variance the Sharpe ratio further improved to 2.9291

Effect of Size on Portfolio Optimization

Table 5: Size effect

	No. of Securities	Expected Returns	Standard Deviation	Sharpe Ratio
Market Portfolio	57	63.78647113	51.8698724	1.229740275
Minimum variance	21	90.13848033	31.98781907	2.817900156
Maximum Return	6	125.195	42.94223	2.9291

The findings in table 5 indicate that reducing the size of assets in the portfolio leads to increased returns and a lower standard deviation with an improved Sharp Ratio. However further reduction of assets number to increases the risk level despite the increase in returns and Sharpe ratio. This operates with the Markowitz theory to the effect that the reduction in the number of assets in the portfolio could compromise optimality of the portfolio. The results therefore affirm that an optimal number of assets is more than 6 assets but less than 21 assets. This is the number that could ensure that the investor attains maximum returns at minimum risk. The same theory argue that increasing the number of assets in a portfolio causes a reduction in the risk levels because then the unsystematic risks are diversified.

Further it was established that the Sharpe portfolio was the most efficient portfolio because it had the highest return. It also concurs with the Modern Portfolio Theory assertion that the higher the risk the higher is the return and investors should be compensated for undertaking high risks in their portfolios.

These findings agree with the works of Newbould and Poon (1993) who affirmed that an optimal portfolio consists of 8 to 20 stocks. Suquaier and Ziyud 2011 also argued that diversification benefits can be obtained when an investor holds 15 to 16 stocks in a portfolio. However, the study by Boscaljion et al (2005) suggested that a randomly selected portfolio of 30 stocks or less selected from industry leaders and equally weighted could provide a well-diversified portfolio. This later study contravene the results in the current study. The argument in both of the above studies agree that increasing portfolio size is worthwhile as long as the marginal benefit of the increased diversification exceeds the marginal cost.

Conclusion

Size had a significant effect on the portfolio optimization, that is, as the number of assets in a portfolio reduce the return from those assets increases significantly. However there is a minimum number beyond which the reduction should not exceed since it will turn around to be risky portfolio. Most scholars have suggested the minimum assets an investor should have in a portfolio must most seem to concur that the minimum number should be 8 and the maximum to be 20 assets. MPT provides that there is need to diversify in a portfolio and the more the assets in a portfolio the better is the return from it. However in reality there must be a maximum number of assets in a given portfolio for an investor to effectively optimize and beyond this number the risk level increase and the return rate reduced. Many scholars assert that the maximum number of assets in a portfolio should be 20 while a few posit the number to be 30.

Empirical finding in this study agree with the scholars who limit the number of assets to be between 8 to 20 assets in a portfolio.

Recommendations

The researcher recommends that investors must always weigh the investment options at their disposal before settling on any specific assets. The specific assets to be chosen should have minimal risks but guarantee high returns. This calls for the need to establish the correlation coefficients among the assets under consideration. Therefore the asset combination chosen should be the one that minimizes the risk involved while maximizing the expected return. This portfolios should have a range of assets between 8 to 20 assets for purposes of realizing the benefits of diversification.

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